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Source Flow Effect on Lineshape

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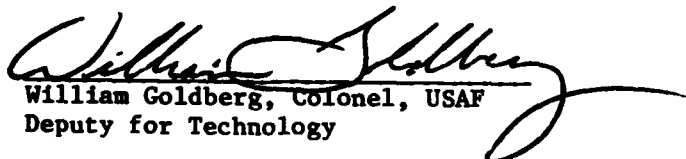
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I. INTRODUCTION

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The effect of particle motion on spectral lineshape (i.e., Doppler broadening) is usually evaluated by assuming that the particles have a random thermal motion.¹ In high-speed gas-flow lasers, of which chemical lasers are an example,² the working fluid may have significant mean velocities as well as random velocities in the optical path direction. In these cases, it is necessary to take the mean motion into account when evaluating the spectral lineshape. The effect of a source flow on Doppler-broadened lineshape is evaluated herein for the case where radiation is perpendicular to the source flow axis, (Fig. 1). The application to a cw chemical laser is then noted.

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¹A. Yariv, Introduction to Optical Electronics, (Holt, Rinehart and Winston, New York, 1971) pp. 76-79.

²D. J. Spencer, H. Mirels, and D. A. Durran, J. Appl. Phys. **43**, 1151 (1972).

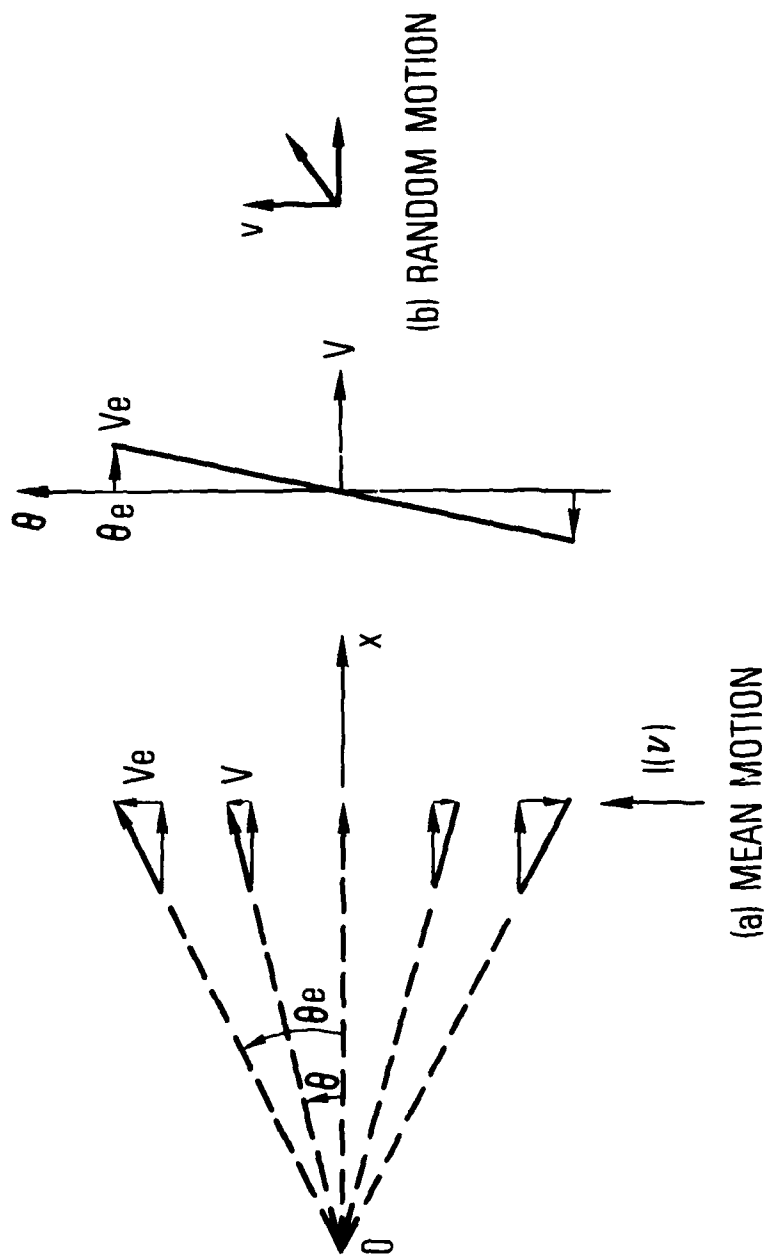


Fig. 1. Source Flow

II. THEORY

Consider a two-dimensional source flow with half angle θ_e (Fig. 1). A radiation field $I(\nu)$ is assumed to propagate in a direction that is transverse to the flow axis. The divergence of the flow results in mean motion in the transverse (optical path) direction. Local mean velocity and random velocity in the transverse direction are denoted by V and v , respectively. The resultant transverse velocity field is $W = V + v$. The radiation frequency ν , which is resonant with particles of velocity W , is found from the Doppler relation¹

$$W = (\nu - \nu_0) c / \nu_0 \quad (1)$$

where ν_0 is the resonant frequency for stationary particles and c is the speed of light. The spectral lineshape is found by determining the distribution function for particles in the range W to $W + dW$, which is found in the following paragraphs.

Consider the case $\theta_e^2 \ll 1$. The flow density and axial flow velocity at each streamwise station x are independent of θ , whereas the transverse velocity can be expressed

$$\frac{V}{V_e} = \frac{\theta}{\theta_e} \quad (2)$$

where V_e is the value of V corresponding to $\theta = \theta_e$. The fraction of particles in the velocity range V to $V + dV$ is

$$\begin{aligned}
F(V) &= 1/(2 v_e) & |v| < v_e \\
&= 0 & |v| > v_e
\end{aligned} \tag{3}$$

Under equilibrium conditions, the fraction of particles in the velocity range v to $v + dv$ has a Maxwellian distribution¹

$$f(v) = \frac{1}{\pi^{1/2} a} e^{-v^2/a^2} \tag{4}$$

where $a \equiv (2 kT/m)^{1/2}$ is the most probable random particle speed. The quantities $F(V)$ and $f(v)$ are normalized so that

$$\int_{-\infty}^{\infty} f(v) dv = \int_{-\infty}^{\infty} F(V) dV = 1 \tag{5a}$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(v) F(V) dv dV = 1 \tag{5b}$$

The integrand in Eq. (5b) denotes the fraction of particles in the combined range V to $V + dV$ and v to $v + dv$. We now replace the independent variables V, v by the pair W, w defined by

$$W = v + V \tag{6a}$$

$$w = v - V \quad (6b)$$

It is seen from Fig. 2 that W, w are orthogonal variables. The particles are confined to the region $-V_e < V < V_e$ in V, v space and to the region $W - 2V_e < w < W + 2V_e$ in W, w space. The particles in the velocity range V to $V + dV$ and v to $v + dv$ are related to the particles in the range W to $W + dW$ and w to $w + dw$ by

$$f(v) F(V) dv dV = f\left(\frac{W-w}{2}\right) F\left(\frac{W+w}{2}\right) \frac{\partial(v, V)}{\partial(w, W)} dw dW \quad (7)$$

where $\partial(v, V)/\partial(w, W)$ is the Jacobian given by³

$$\frac{\partial(v, V)}{\partial(w, W)} = \begin{vmatrix} v_w & v_W \\ w_w & w_W \end{vmatrix} = \frac{1}{2} \quad (8)$$

Equation (5b) becomes

$$1 = \int_{-\infty}^{\infty} dW \int_{W-2V_e}^{W+2V_e} dw \left(\frac{e^{-(w+W)^2/(4a^2)}}{4\pi^{1/2} a V_e} \right) \quad (9a)$$

$$= \int_{-\infty}^{\infty} dW \left[\frac{\operatorname{erf}\left(\frac{W+V_e}{a}\right) - \operatorname{erf}\left(\frac{W-V_e}{a}\right)}{4V_e} \right] \quad (9b)$$

The integrand in Eq. (9b) can be interpreted as the fraction of particles in the range W to $W + dW$. Introduce the notation $X_e \equiv V_e/a$, and

³D. H. Menzel, Fundamental Formulas of Physics, Vol. I, (Dover Publications, New York, 1960), p. 26.

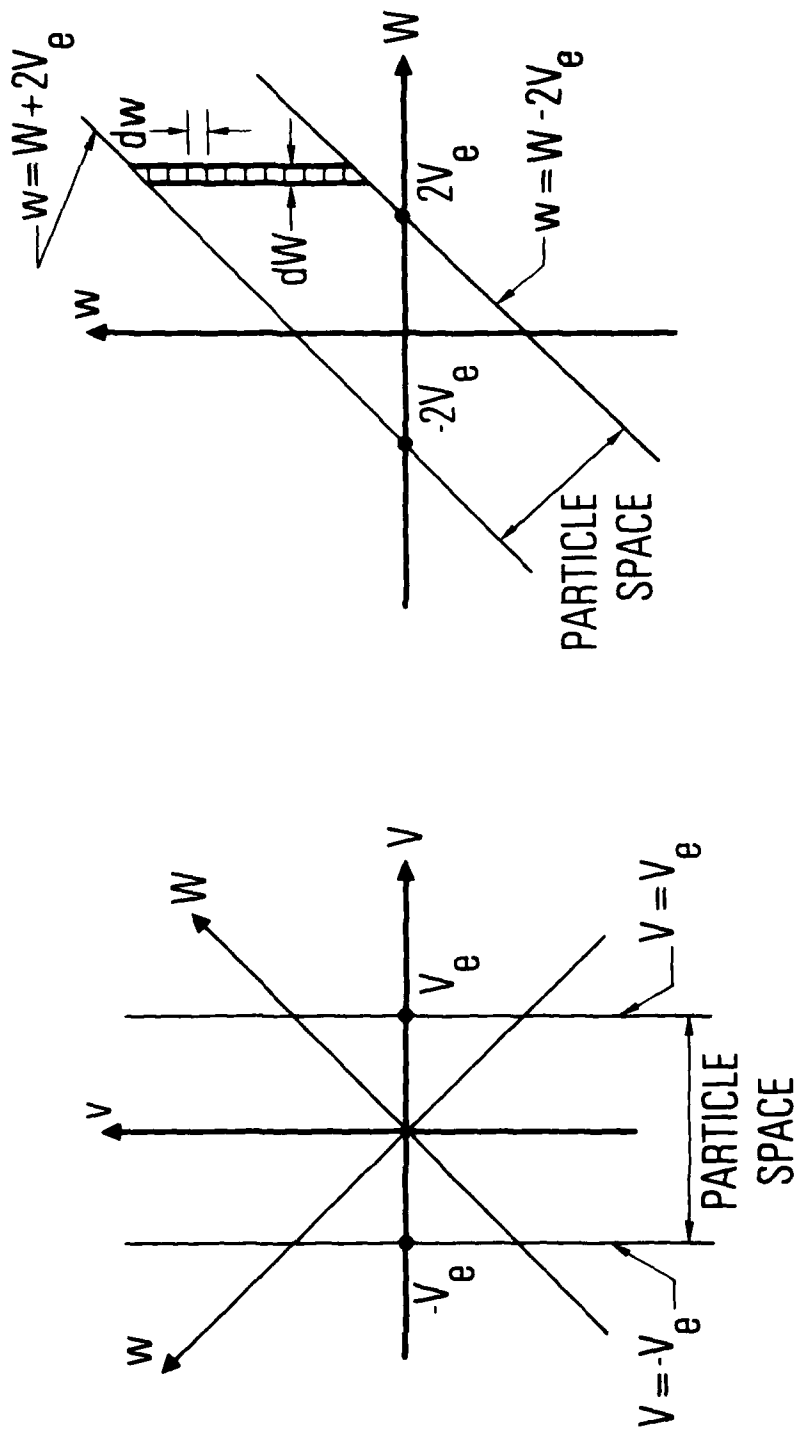


Fig. 2. Particle Space in V - v and W - w Coordinates

$$X \equiv \frac{W}{a} \equiv \frac{v-v_0}{v_0} \frac{c}{a} \equiv 2(\ln 2)^{1/2} \frac{v-v_0}{\Delta v_D} \quad (10)$$

where Δv_D is the Doppler width (FWHM) corresponding to $X_e = 0$. The fraction of particles in the interval X to $X + dX$ is denoted $n(X, X_e)$ and, from Eq. (9b), equals

$$n(X, X_e) = [\text{erf}(X + X_e) - \text{erf}(X - X_e)] / (4X_e) \quad (11)$$

Limiting forms of $n(X, X_e)$ are

$$\pi^{1/2} n(X, X_e) = 1 - \left(\frac{X_e^2}{3} + X^2 \right) + \left(\frac{X_e^4}{10} + X_e^2 X^2 + \frac{X^4}{2} \right) - \dots \quad (12a)$$

$$= e^{-X^2} [1 + O(X_e^2)] \quad (12b)$$

$$= \frac{\pi^{1/2}}{2X_e} \text{erf}(X_e) [1 + O(X^2)] \quad (12c)$$

Thus, transverse mean flow effects are negligible for $(X_e)^2 \ll 1$ and must be considered for $(X_e)^2 > 0(1)$. This result is physically realistic, since X_e is the ratio of characteristic mean to characteristic random motion in the transverse direction. The variation of $\pi^{1/2} n(X, X_e)$ with X is plotted in Fig. 3 for various values of X_e . Figure 3 can be used directly to estimate the effect of

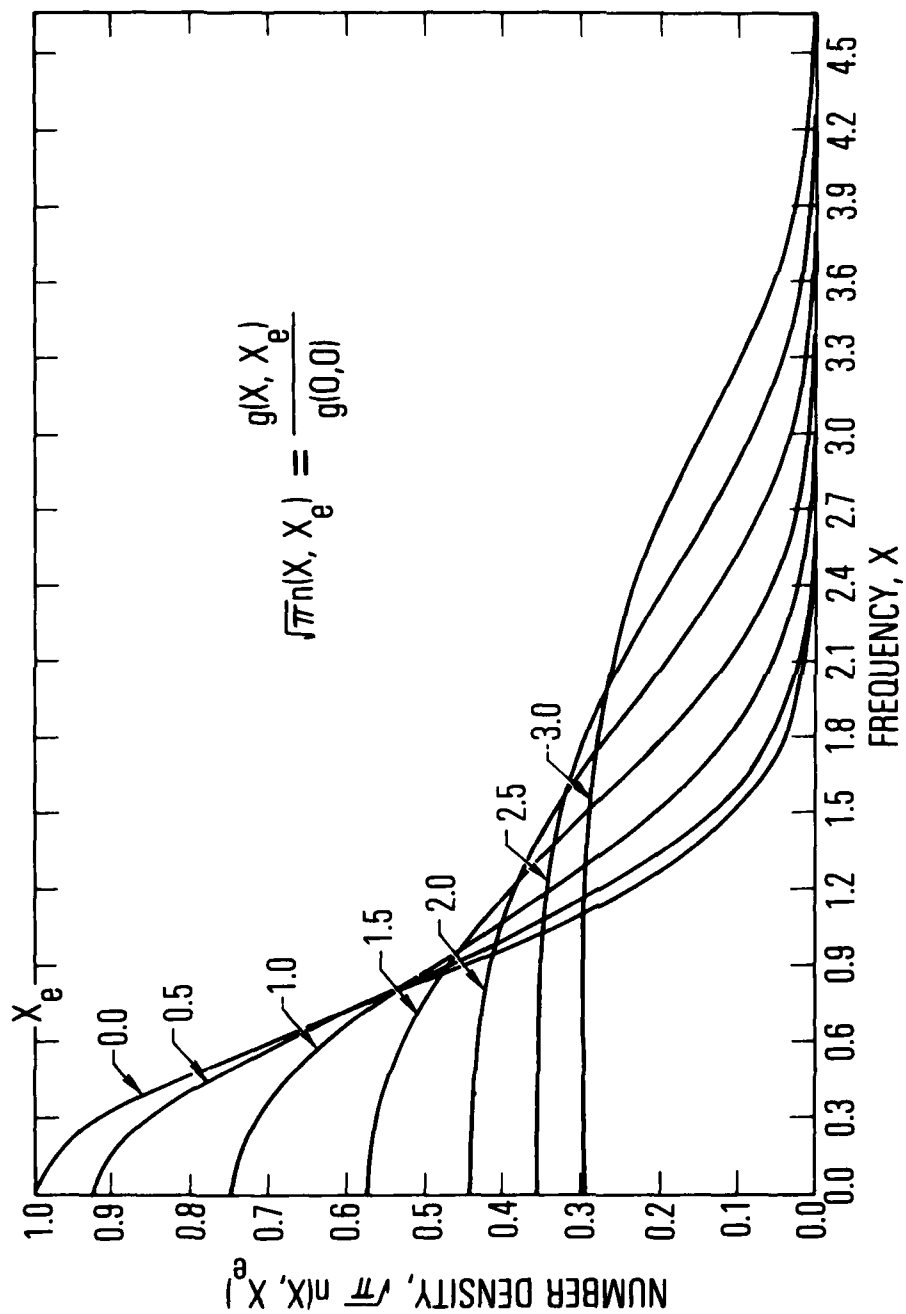


Fig. 3. Lineshape [Eqs. (11) and (13)]

transverse mean motion on zero power gain of a Doppler-broadened medium. If we let $g_0(X, X_e)$ denote the zero power gain corresponding to X, X_e , it follows that in the Doppler limit^{4,5} (i.e., in the limit of $\Delta v_h / \Delta v_D \ll 1$, where Δv_h is the homogeneous half-width)

$$\frac{g_0(X, X_e)}{g_0(0,0)} = \pi^{1/2} n(X, X_e) \quad (13)$$

where $g_0(0,0)$ is the line center value in the absence of transverse motion. Thus, the ordinate in Fig. 3 is a direct measure of the effect of transverse mean motion on zero power gain. Line center gain is decreased, and the lineshape is broadened as X_e is increased. The corresponding value of the anomalous index of refraction $n(X, X_e)$ can be expressed,^{4,5} for $\lambda \equiv c/v_0$,

$$\frac{2\pi}{\lambda} \frac{n(X, X_e) - 1}{g(0,0)} = \frac{1}{8\pi^{1/2} X_e} \int_{-\infty}^{\infty} \frac{dX_o}{X - X_o} [\text{erf}(X_o + X_e) - \text{erf}(X_o - X_e)] \quad (14a)$$

$$= \frac{1}{\pi^{1/2}} \frac{X}{X_e} D(X_e) [1 + O(X^2)] \quad (14b)$$

$$= \frac{1}{\pi^{1/2}} D(X) [1 + O(X_e^2)] \quad (14c)$$

⁴H. Mirels, AIAA J. 17 (5), 478 (1979)

⁵M. Sargent, M. O. Scully II, and W. E. Lamb, Jr., Laser Physics, (Addison-Wesley, Massachusetts, 1974), pp. 144-155.

where $D()$ is the Dawson integral. Equation (14a) is plotted in Fig. 4. The maximum value of the index decreases as X_e increases and occurs at values of X and X_e related by $D(X + X_e) = D(X - X_e)$, which for $(X + X_e)^{-2} \ll 1$ becomes $X^2 = X_e^2 + 0.5 [1 + O(X + X_e)^{-2}]$.

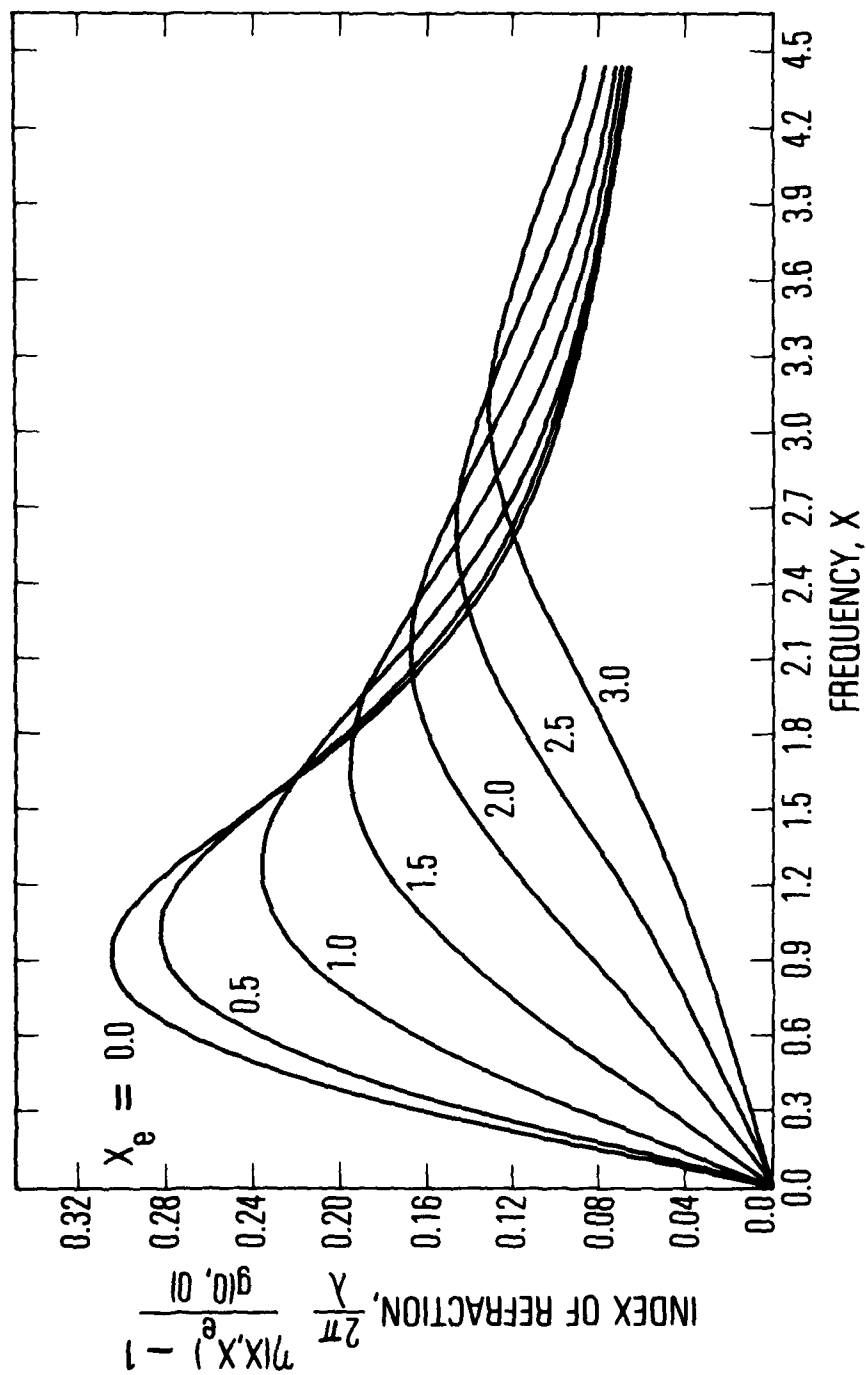


Fig. 4. Anomalous Index of Refraction [Eq. (14)]

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III. APPLICATION

It is convenient to express the parameter X_e in terms of local flow Mach number. The most probable speed a is related to the local speed of sound $a_s = (\gamma RT/m)^{1/2}$ by the relation $a_s/a = (\gamma/2)^{1/2}$ where γ is the ratio of specific heats, which equals 7/5 and 5/3 for diatomic and monatomic gases, respectively. Let $M_e \equiv V_e/a_s$ denote the characteristic transverse flow Mach number, and it follows that X_e is related to M_e by

$$X_e = (2/\gamma)^{1/2} M_e \quad (15)$$

Thus, X_e is nearly equal to the characteristic transverse flow Mach number.

The present results can be applied to cw chemical lasers in which transverse flow expansion is permitted. Typical axial flow Mach numbers² are in the range 4 to 6 and can be characterized as being of order 5. The corresponding value of X_e is then of order $X_e = 0(5\theta_e)$. Thus, transverse flow effects are negligible for $\theta_e^2 \ll 0(1/25)$ and must be considered for $\theta_e^2 > 0(1/25)$.

The effect of source flow on cw chemical laser output power is being investigated for the case of a two-level model,⁴ laminar mixing, a Fabry Perot resonator, and multiple longitudinal modes spacing [i.e., $\Delta\nu_c/\Delta\nu_h \ll (1)$, where $\Delta\nu_c = c/2L$ is the longitudinal mode spacing]. Preliminary results are indicated in Fig. 5. The ordinate P/P_{sat} is the ratio of output power to saturated output power, and the abscissa is the ratio of cavity threshold gain, g_c , to the maximum zero power gain, g_{mzp} , for the case $X_e = 0$. The decrement in output power caused by source flow is seen to become more severe as threshold gain is increased.

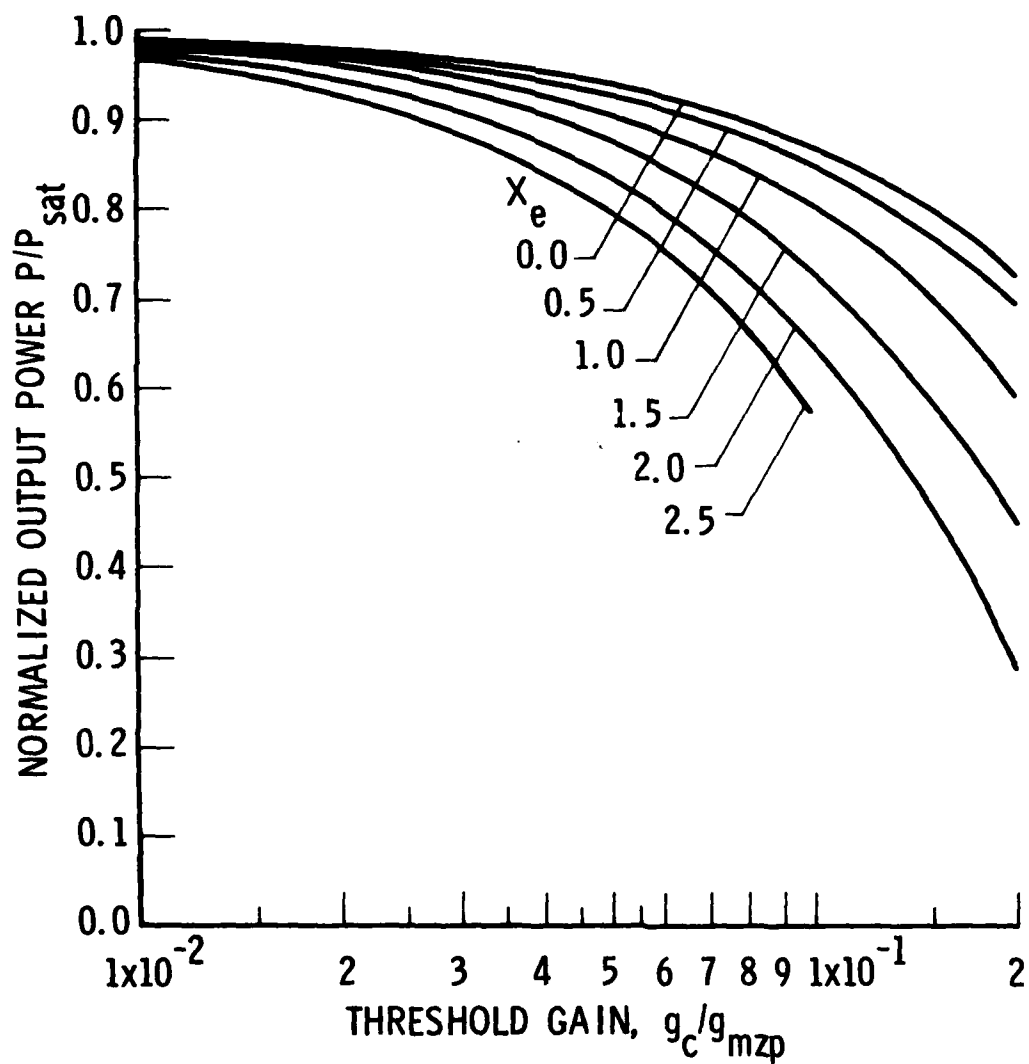


Fig. 5. Effect of Threshold Gain and Source Flow on Output Power from Multiple Longitudinal Mode CW Chemical Laser Employing a Fabry - Perot Resonator

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